MATH 135 — QUIZ 7 SOLUTIONS — JAMES HOLLAND
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Question 1. Suppose a shephard with s sheep requires (in total) $C(s) = 5s^2 - 8\sqrt{s+1}$ dollars per month care for them. Using marginal analysis, estimate the cost of the 16th sheep.

Solution .:.

For $s \approx 15$, we have that $C(s) \approx C(15) + C'(15) \cdot (s - 15)$. So the cost of the 16th sheep is

$$C(16) - C(15) \approx [C(15) + C'(15) \cdot 1] - C(15) = C'(15).$$

Since $C'(s) = 10s - 4(s+1)^{-1/2}$, we have that the cost of the 16th sheep is roughly $C'(15) = 150 - 4 \cdot \frac{1}{4} = 149$ dollars per month.

Question 2. Two people are tied together by a 10 foot long rope. Suppose one of the people is asleep on the ground, 8 feet away from the side of a building. The other person is 6 feet high, climbing the side of the building at a rate of 1 foot per second, and so dragging the sleeping person along the ground. How fast will the sleeper be dragged after 2 seconds?

Solution .:.

The situation describes a right-triangle with a hypotenuse length of 10 feet. The vertical side of the triangle has a length equal to the height of the climber, $H(t)$, which is given by $H(t) = 6_{\mathrm{ft}} + 1_{\mathrm{ft/s}} t$, for t measured in seconds. The distance of the sleeping person from the side of the building can then be calculated using the Pythagorean theorem:

$$H(t)^2 + D(t)^2 = (10_{\mathrm{ft}})^2.$$

Using implicit differentiation to find $\frac{\mathrm{d}D}{\mathrm{d}t}$, we have that $2H(t)\frac{\mathrm{d}H}{\mathrm{d}t} + 2D(t)\frac{\mathrm{d}D}{\mathrm{d}t} = 0$, and hence using the values given in the problem,

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{-2H(t)\frac{\mathrm{d}H}{\mathrm{d}t}}{2D(t)} = \frac{-H(t)\frac{\mathrm{d}H}{\mathrm{d}t}}{D(t)} = \frac{-\left(6_{\mathrm{ft}} + 1_{\mathrm{ft/s}} \cdot 2_{\mathrm{s}}\right) \cdot 1_{\mathrm{ft/s}}}{D(2_{\mathrm{s}})} = \frac{-8_{\mathrm{ft}^2/\mathrm{s}}}{D(2_{\mathrm{s}})}.$$

To find $D(2_{\mathrm{s}})$, we note that $H(2_{\mathrm{s}})^2 + D(2_{\mathrm{s}})^2 = (10_{\mathrm{ft}})^2$ implies that $(8_{\mathrm{ft}})^2 + D(2_{\mathrm{s}})^2 = 100_{\mathrm{ft}^2}$ and hence $D(2_{\mathrm{s}}) = \sqrt{36_{\mathrm{ft}^2}} = 6_{\mathrm{ft}}$. Therefore,

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{-8_{\mathrm{ft}^2/\mathrm{s}}}{6_{\mathrm{ft}}} = \left(-\frac{4}{3}\right)_{\mathrm{ft/s}}.$$

Question 3. Consider the function $f(x) = x^2 - 2x - 15$.
The actual value of $f(4.9) = -0.79$.
Using methods learned in class, estimate $f(4.9)$.
NOTE: do not write '=' when you mean '≈' (nor '≈' when you mean '=').

Solution .:.

For $x \approx 5$, $f(x) \approx f(5) + f'(5) \cdot (x - 5)$. In particular, since $f(5) = 25 - 10 - 15 = 0$, and $f'(x) = 2x - 2$, we have that

$$f(x) \approx 0 + (2 \cdot 5 - 2) \cdot (x - 5) = 8(x - 5) \quad \text{so that} \quad f(4.9) \approx 8 \cdot (-0.1) = -0.8,$$

which is a pretty good approximation for the actual value of -0.79.